



# Mathematical and Computational Oncology

## *Framework and tools for predictive systems*

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# Overview

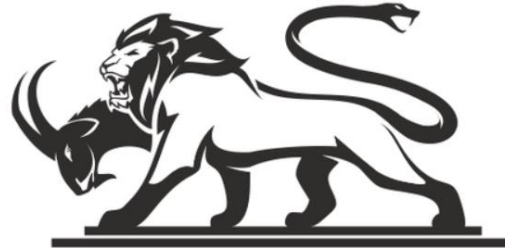
A Framework for Mathematical and Computational Oncology



**GLUECK**



**TUCANN**



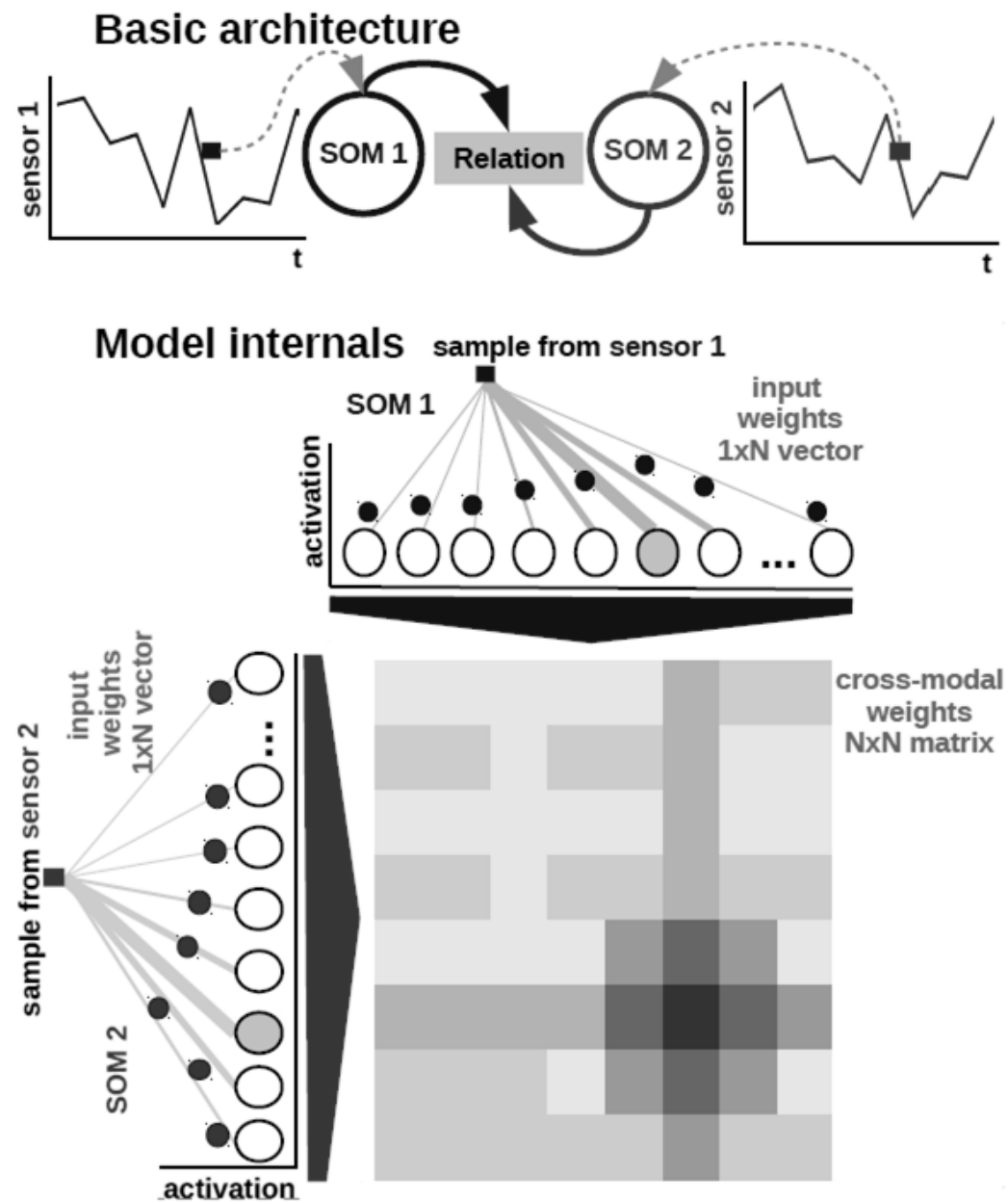
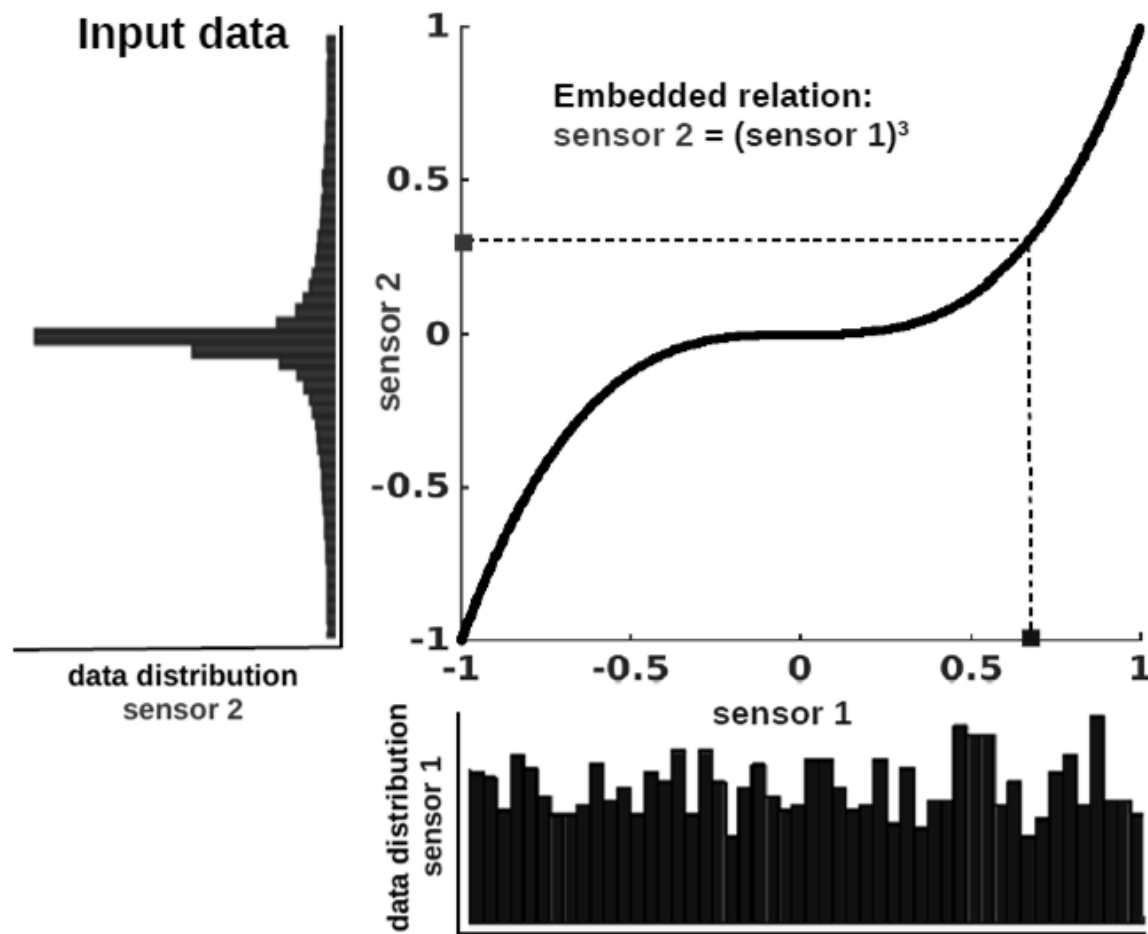
**CHIMERA**



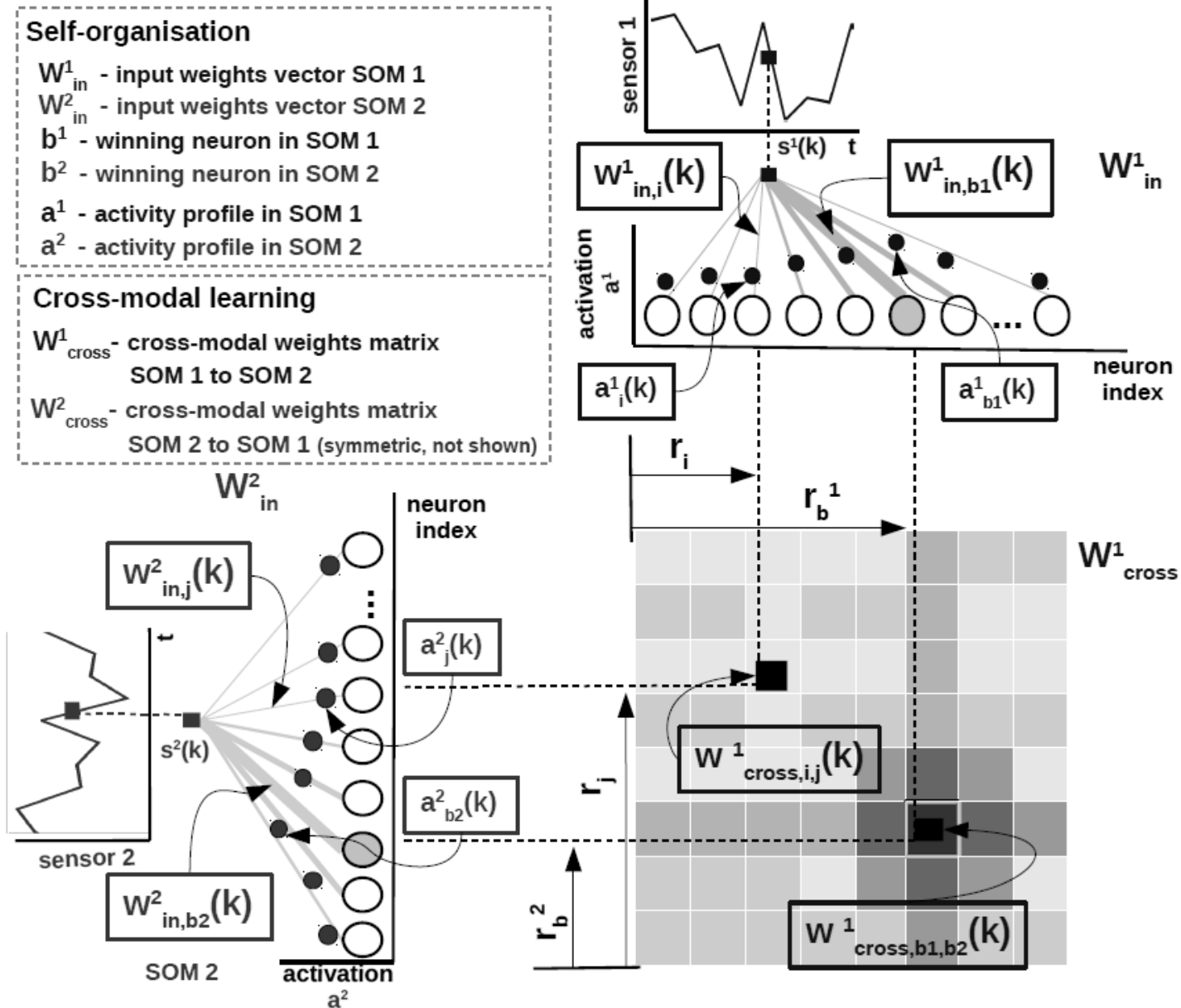
**PERFECTO**

# A Framework for Mathematical and Computational Oncology

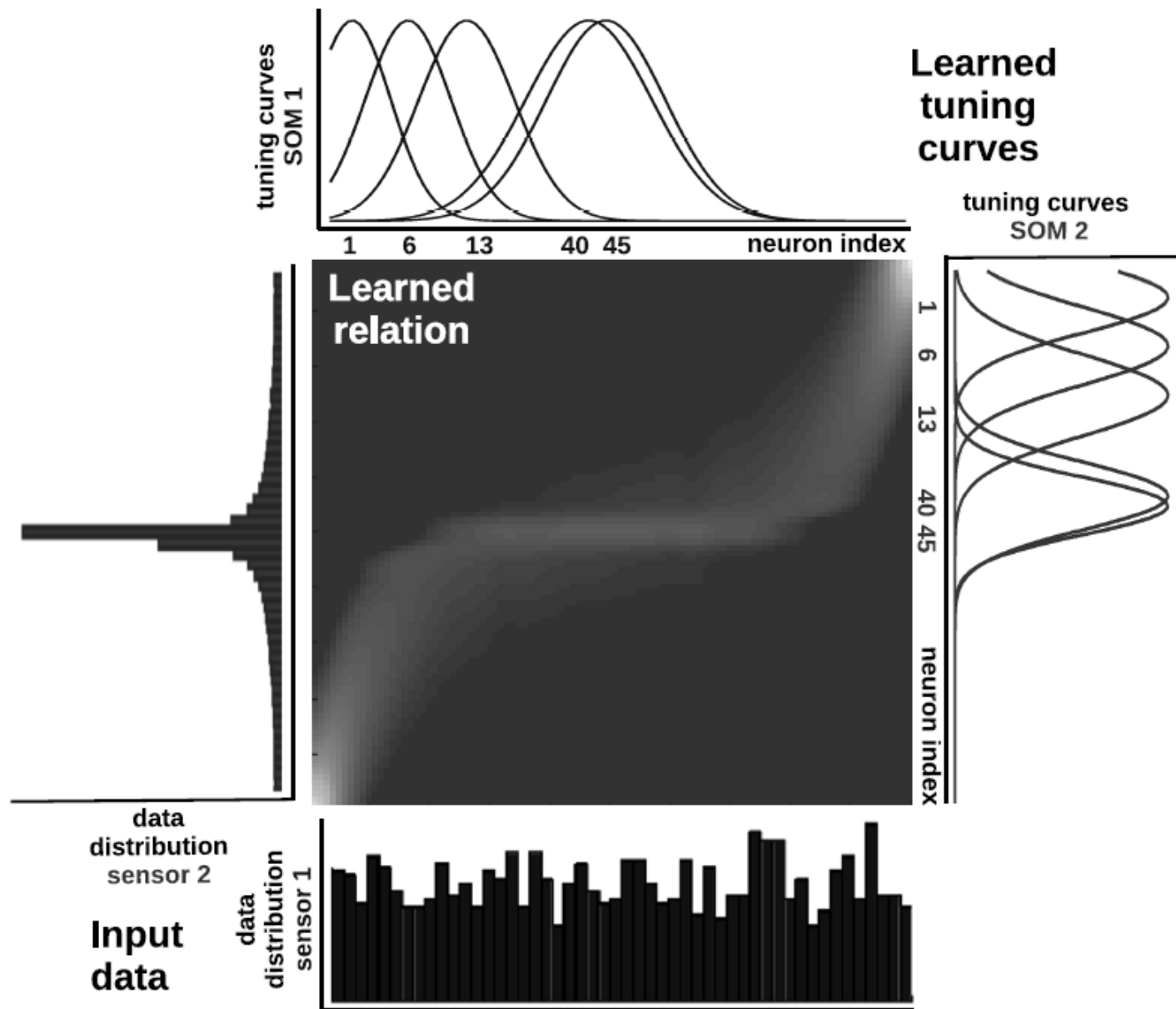
# Core model



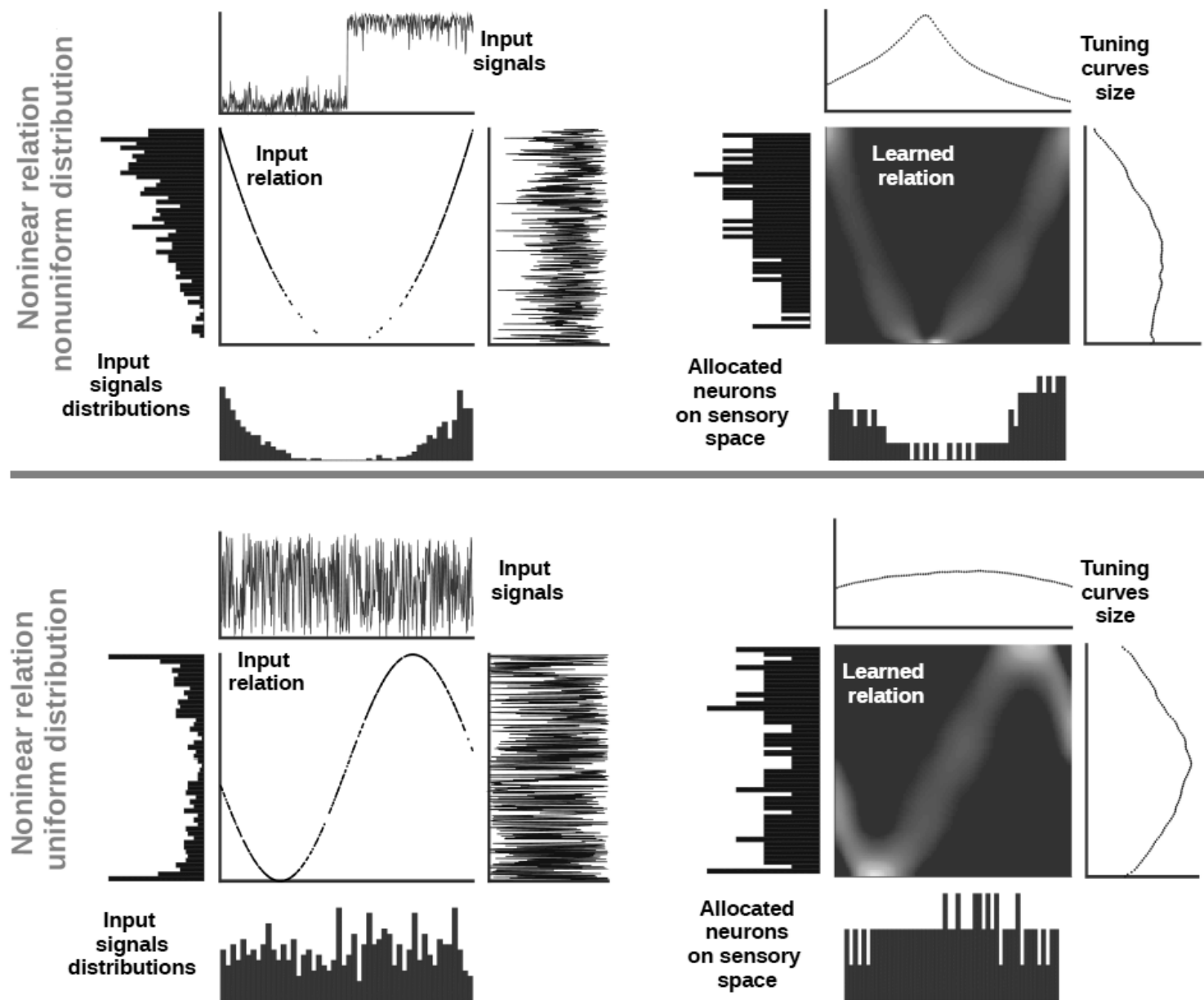
# Core model internals



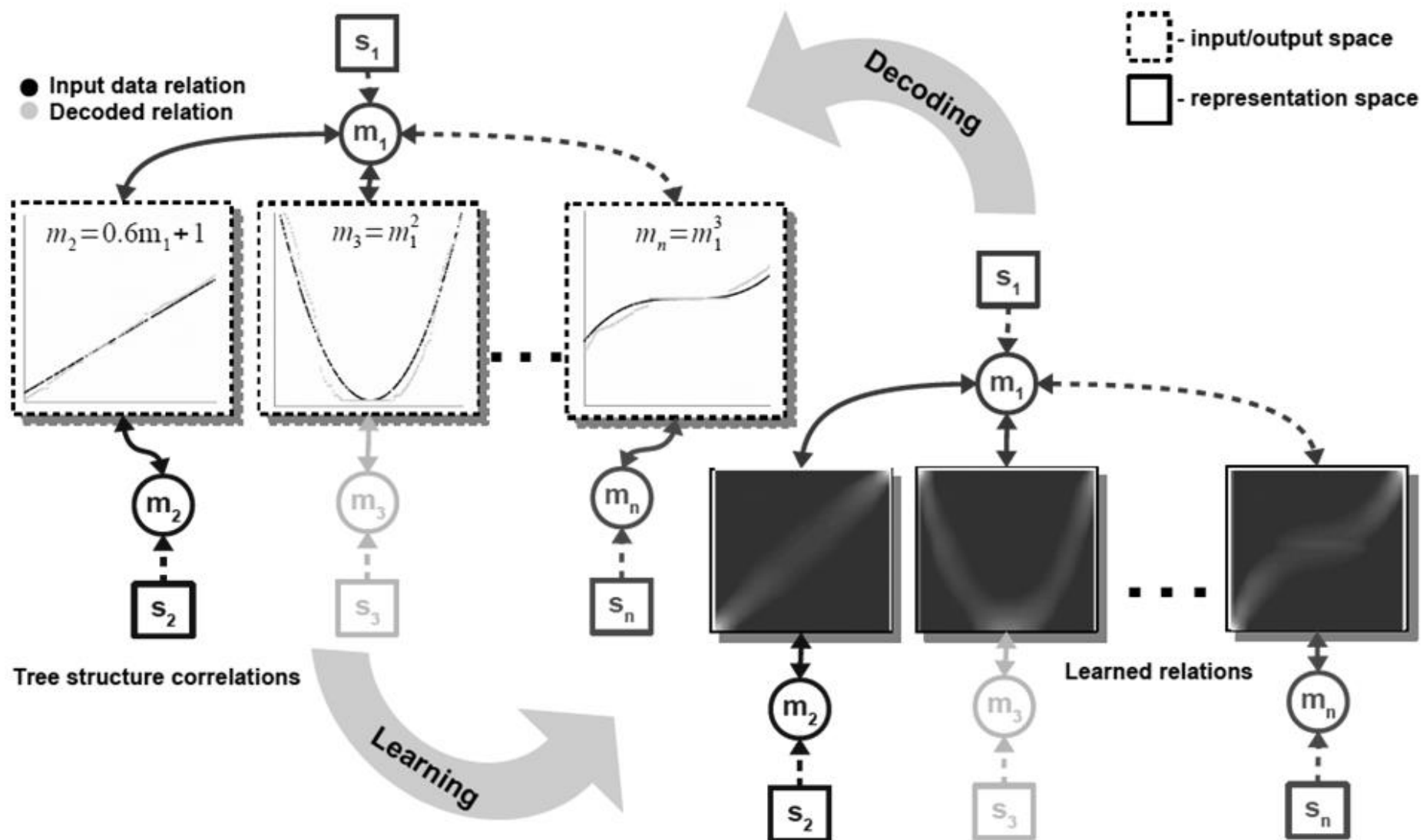
# Learning capabilities I



# Learning capabilities II

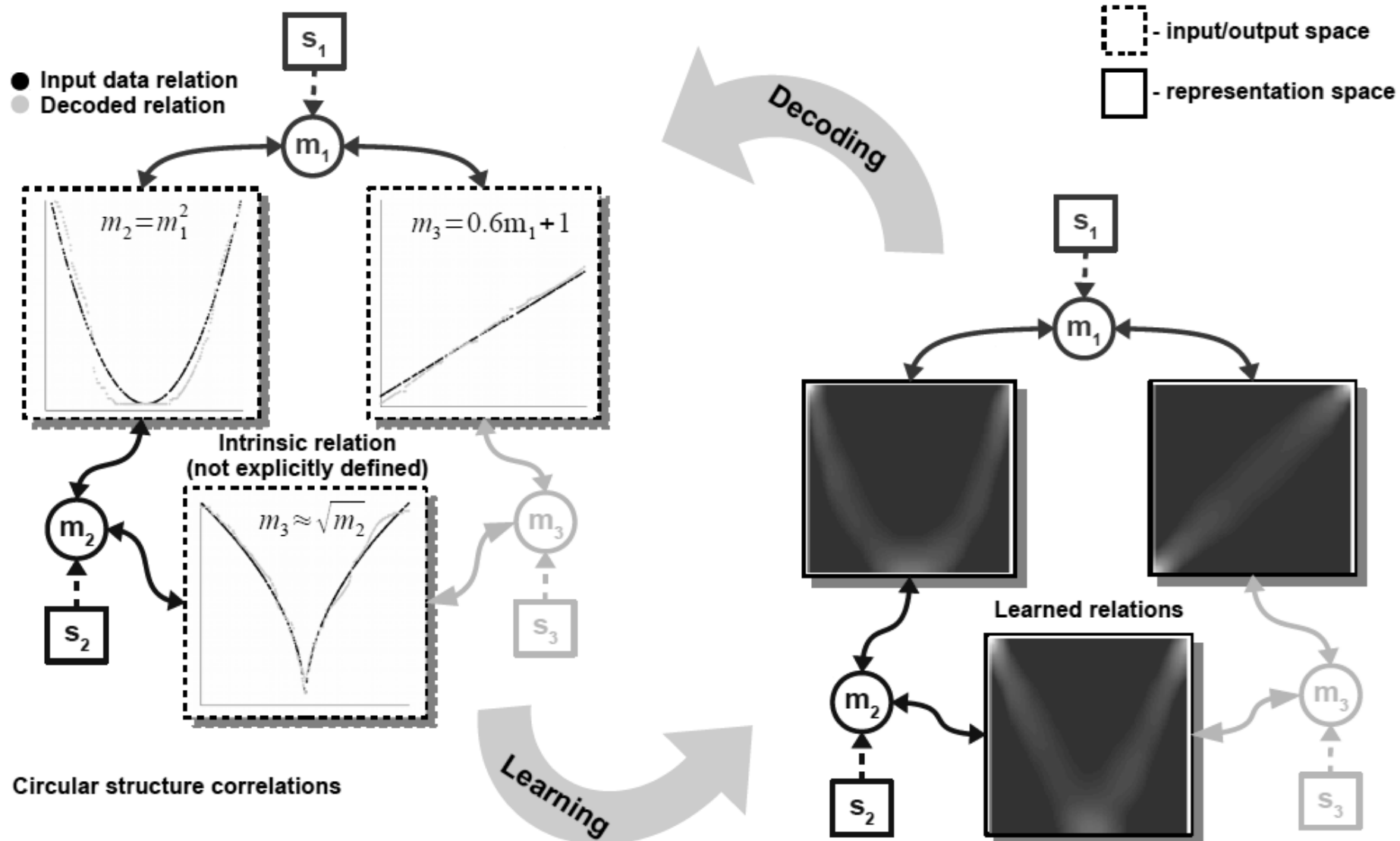


# Extensibility I





# Extensibility II

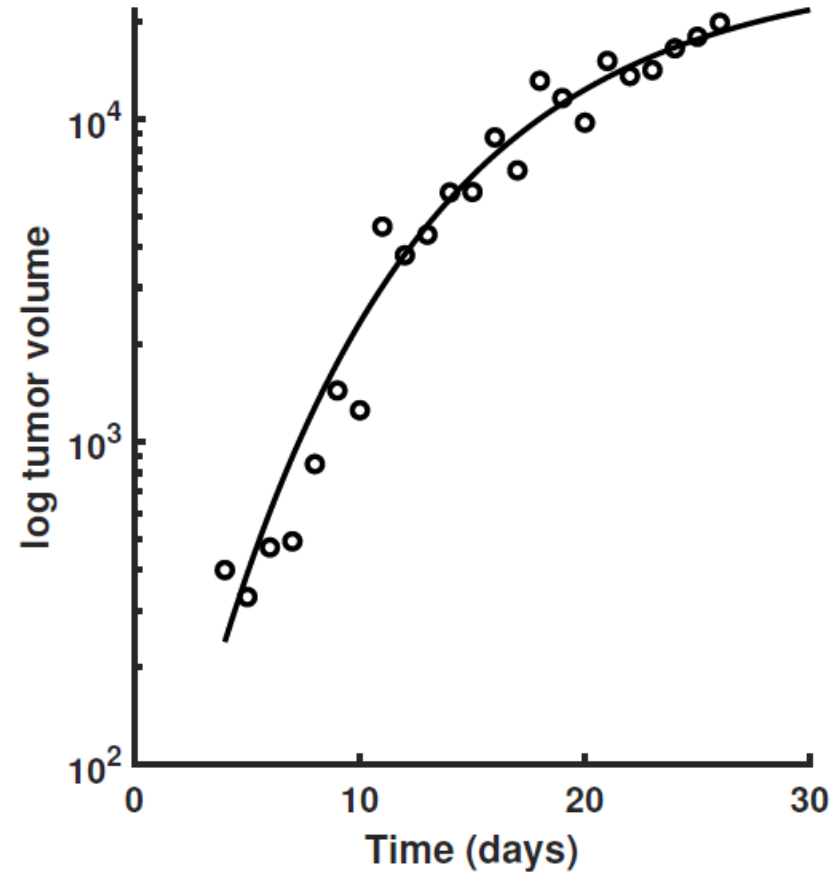
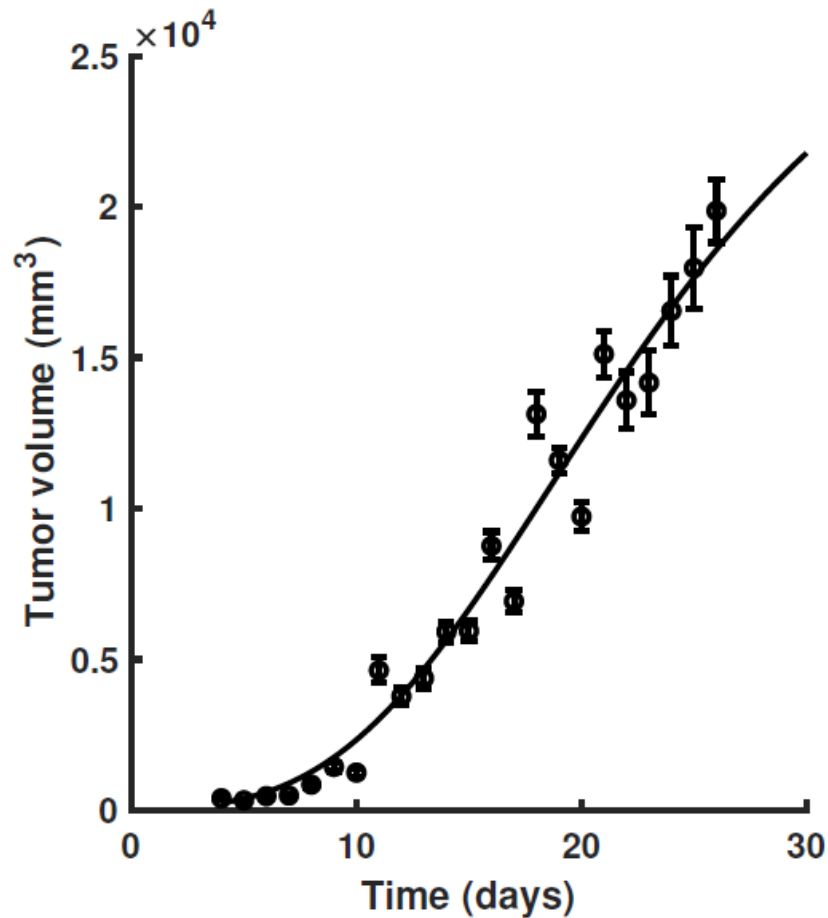




**GLUECK**

**G**rowth pattern **L**earning for  
**U**nsupervised **E**xtraction of **C**ancer **K**inetics

# Tumor growth data

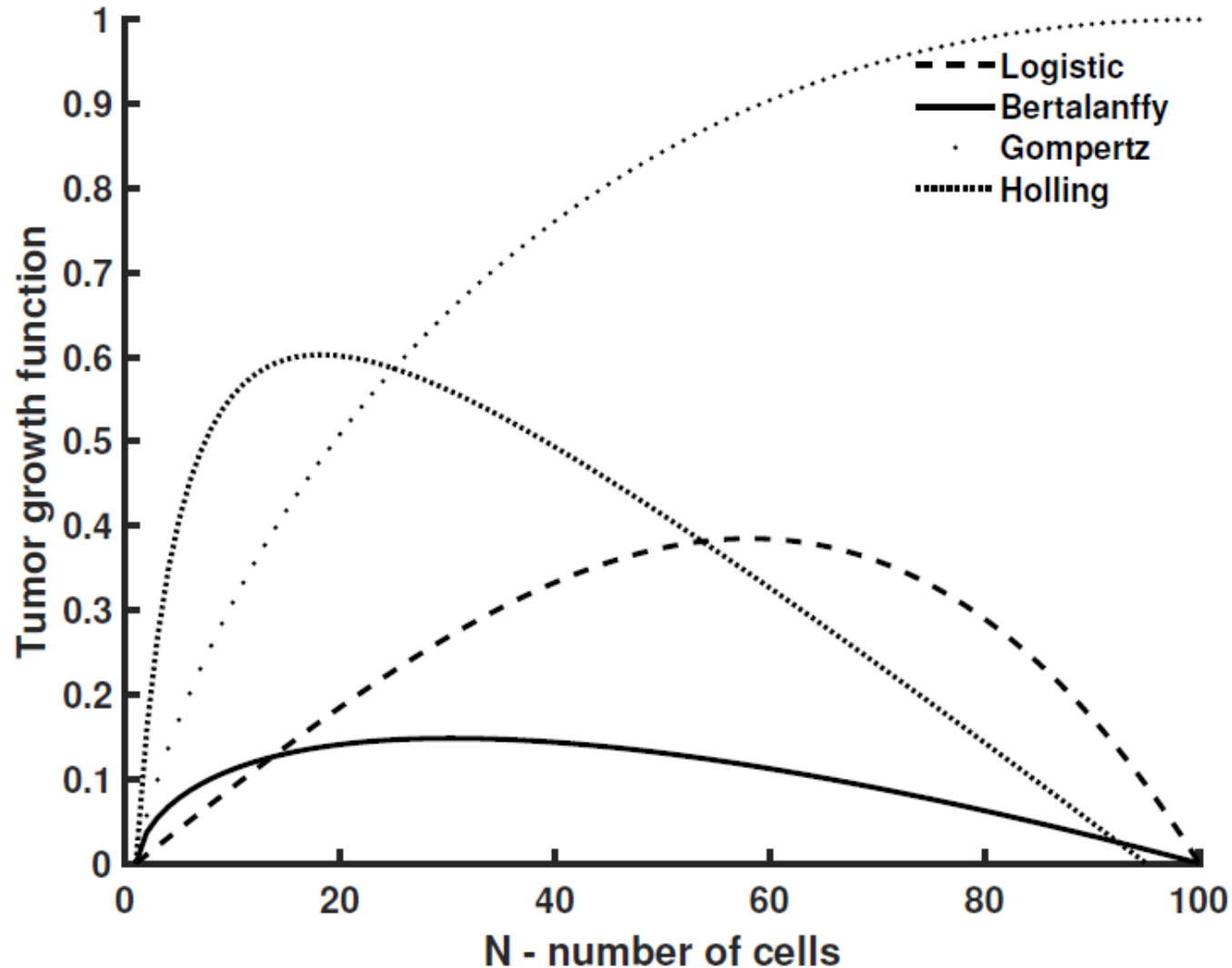


Peculiarities of data:

- Small
- Unevenly sampled
- High-variability
- Heterogeneous
- Model selection is hard
- Determines treatment

Growth kinetics of Fortner Plasmacytoma 1 tumors. Points represent mean volume of subcutaneous tumor implants in mice, error bars represent  $\pm 1$  standard error of the mean at each point. Data from Simpson-Herren et al. Cancer Chemother Rep 54(3)

# Tumor growth models

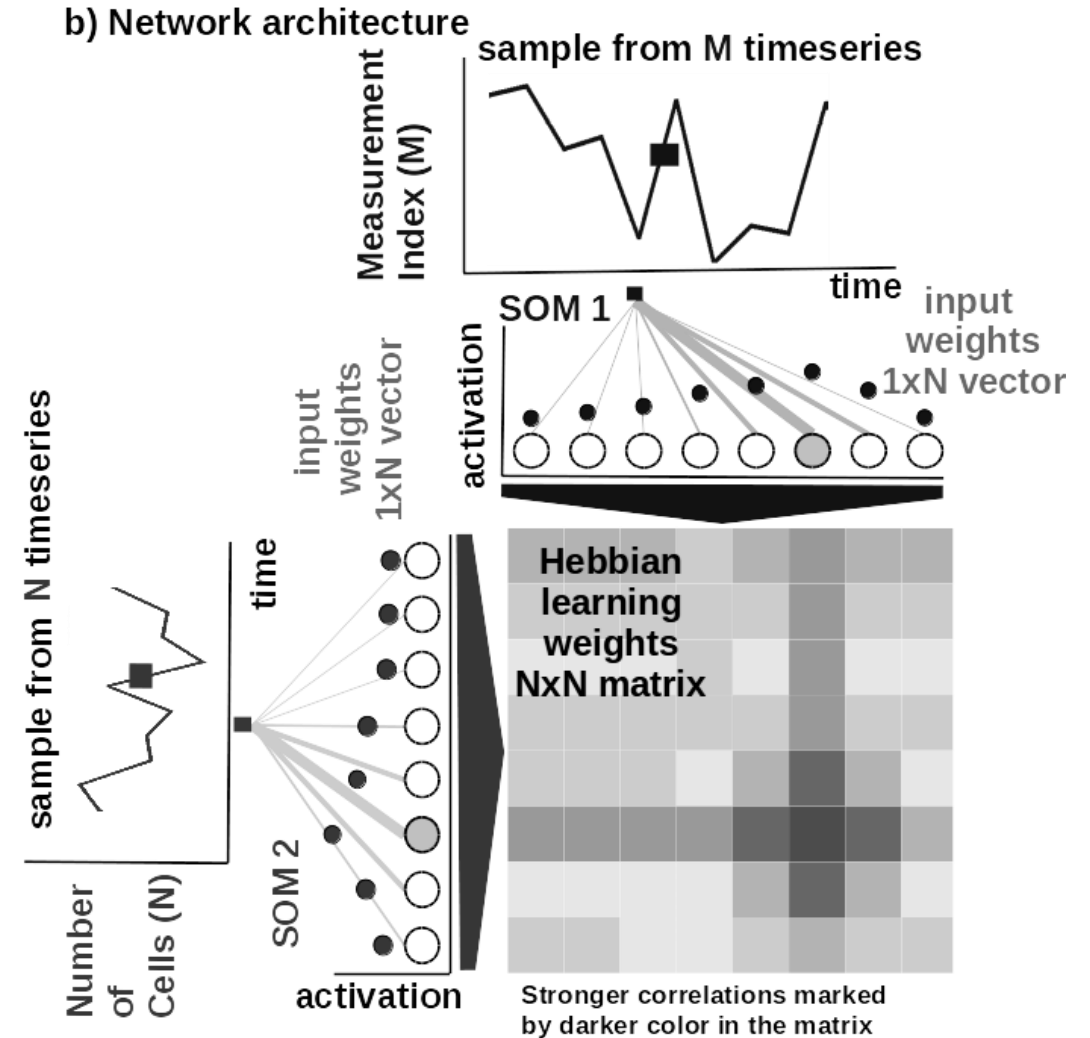
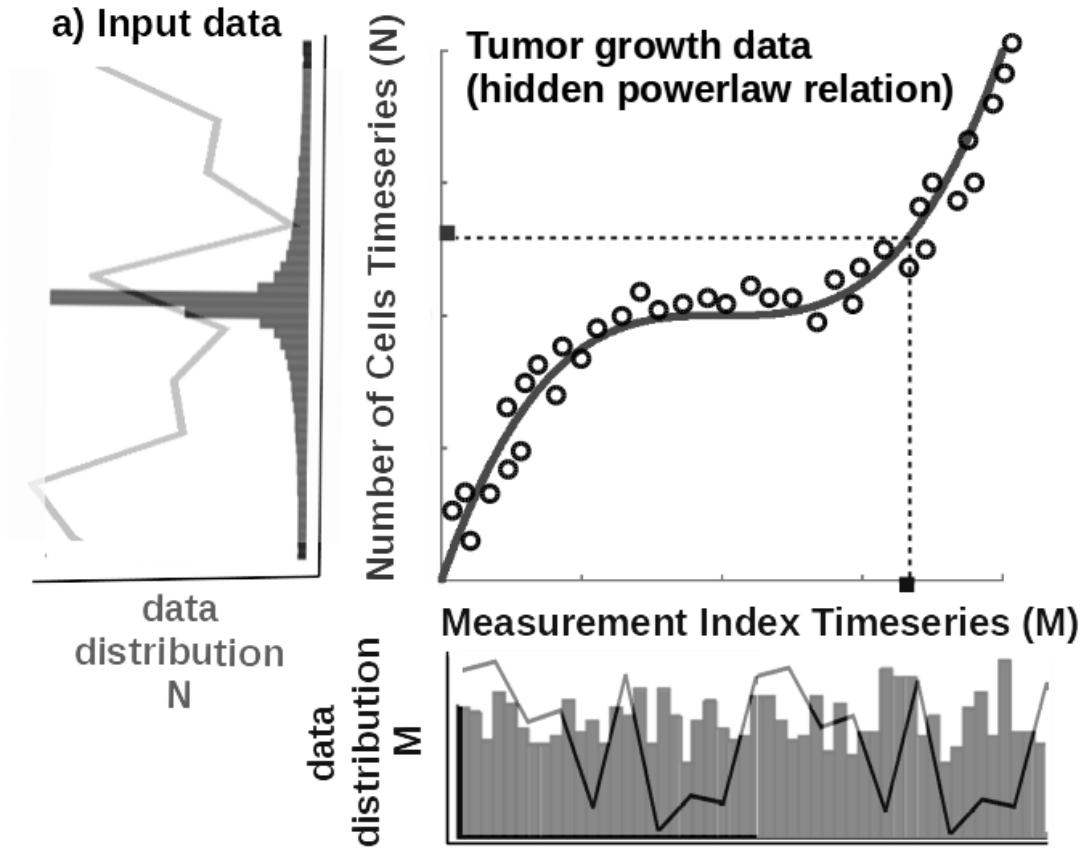


Model	Equation
Logistic	$\frac{dN}{dt} = \alpha N - \beta N^2$
Bertalanffy	$\frac{dN}{dt} = \alpha N^\lambda - \beta N$
Gompertz	$\frac{dN}{dt} = N(\beta - \alpha \ln N)$
Holling	$\frac{dN}{dt} = \frac{\alpha N}{k+N} - \beta N$

Parameters:

$N$  - cell population size (or volume),  
 $\alpha$  - growth rate,  
 $\beta$  - cell death rate,  
 $\lambda$  - nutrient limited proliferation rate,  
 $k$  - carrying capacity of cells.

# Instantiating the model



# Experiments and evaluation

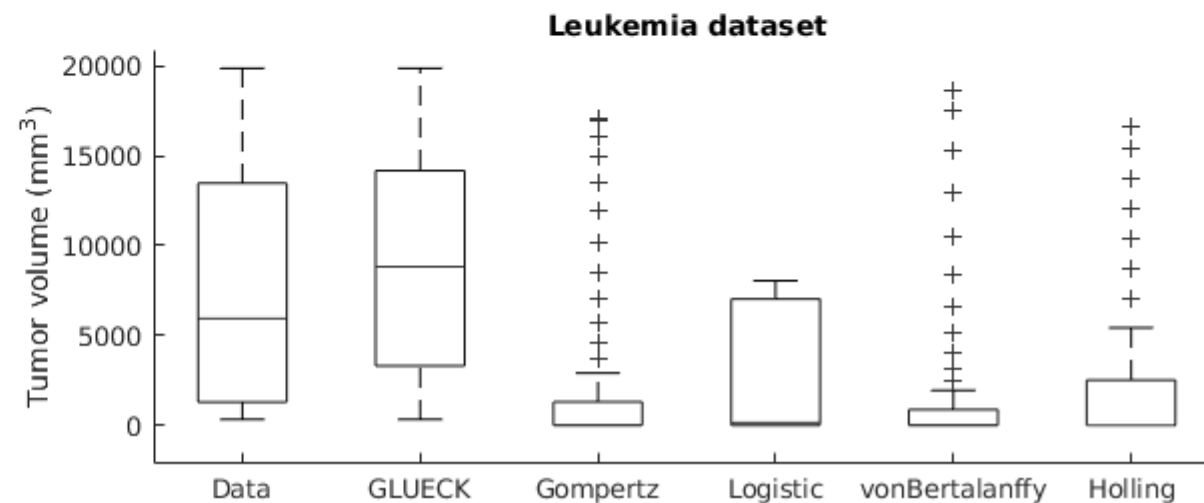
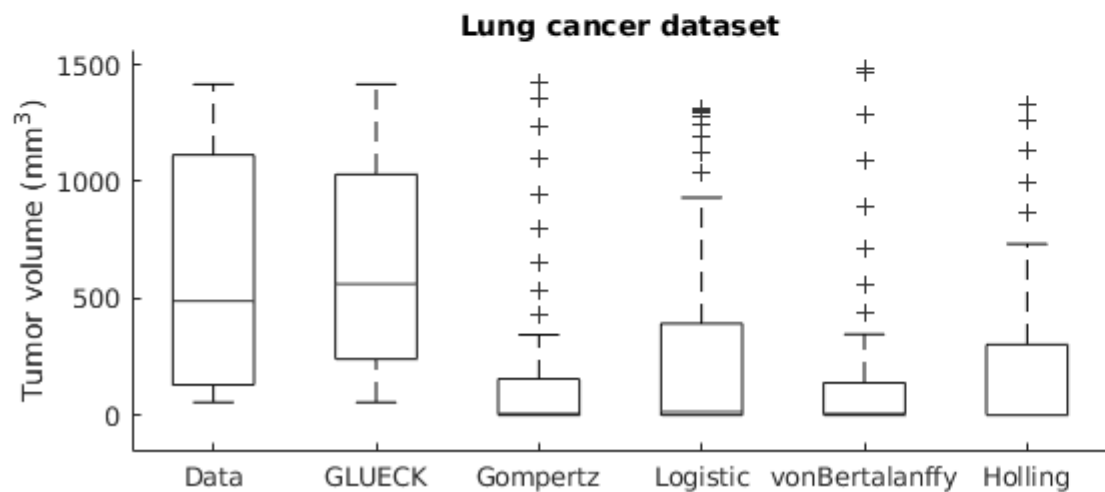
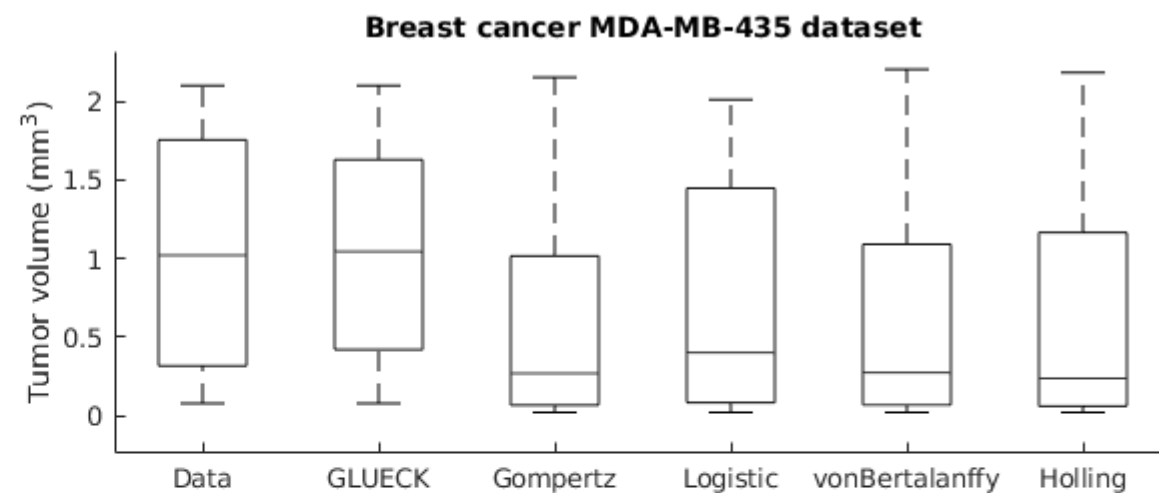
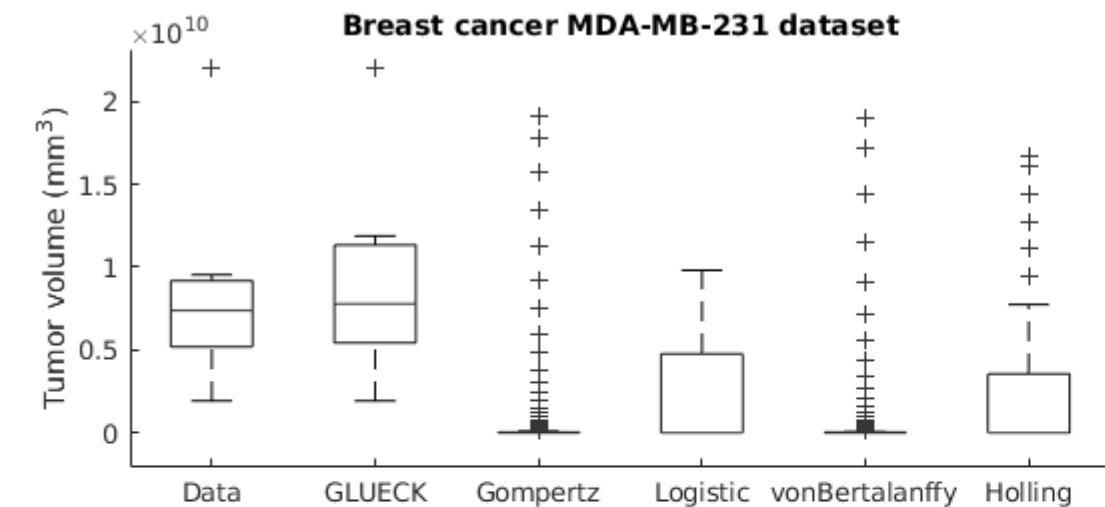
Experimental dataset setup				
Dataset	Cancer Type	Data Type	Data Points	Data Freq.
1	Breast (MDA-MB-231)	Fluorescence imaging	7	2x/week
2	Breast (MDA-MB-435)	Digital Caliper	14	2x/week
3	Lung	Caliper	10	7x/week
4	Leukemia	Microscopy	23	7x/week

Metric	Equation
SSE	$\sum_{i=1}^N (\frac{y^i - y_m^i}{\sigma_i})$
RMSE	$\sqrt{\frac{SSE}{N-p}}$
sMAPE	$\frac{1}{N} \sum_{i=1}^N (2 \frac{ y^i - y_m^i }{( y^i  +  y_m^i )})$
AIC	$N \ln(\frac{SSE}{N}) + 2p$
BIC	$N \ln(\frac{SSE}{N}) + \ln(N)p$

Evaluation metrics for tumor growth models.

We consider:  
 $N$  - number of measurements,  
 $\sigma$ - standard deviation of data,  
 $p$  - number of parameters of the model.

# Experiments and evaluation



# Experiments and evaluation

Evaluation Metrics (smaller value is better)							
Dataset/Model	SSE	RMSE	sMAPE	AIC	BIC	Rank <sup>a</sup>	
<i>Breast<sup>b</sup> cancer</i> <a href="#">[20]</a>							
Logistic	7009.6	37.4423	1.7088	52.3639	52.2557	<b>2</b>	
Bertalanffy	8004.9	44.7350	1.7088	55.2933	55.1310	<b>5</b>	
Gompertz	7971.8	39.9294	1.7088	53.2643	53.1561	<b>4</b>	
Holling	6639.1	40.7403	1.4855	53.9837	53.8215	<b>3</b>	
<b>GLUECK</b>	119.3	4.1285	0.0768	19.8508	19.8508	<b>1</b>	
<i>Breast<sup>c</sup> cancer</i> <a href="#">[26]</a>							
Logistic	0.2936	0.1713	0.1437	-40.5269	-39.5571	<b>4</b>	
Bertalanffy	0.2315	0.1604	0.1437	-41.3780	-39.9233	<b>2</b>	
Gompertz	0.3175	0.1782	0.1437	-39.5853	-38.6155	<b>5</b>	
Holling	0.2699	0.1732	0.1512	-39.5351	-38.0804	<b>3</b>	
<b>GLUECK</b>	0.0977	0.0902	0.0763	-57.7261	-57.7261	<b>1</b>	

<sup>a</sup> Calculated as best in 3/5 metrics.

<sup>b</sup> MDA-MB-231 cell line

<sup>c</sup> MDA-MB-435 cell line

Evaluation Metrics (smaller value is better)							
Dataset/Model	SSE	RMSE	sMAPE	AIC	BIC	Rank <sup>a</sup>	
<i>Lung cancer</i> <a href="#">[6]</a>							
Logistic	44.5261	2.2243	1.5684	19.3800	20.1758	<b>2</b>	
Bertalanffy	54.1147	2.6008	1.5684	23.5253	24.7190	<b>5</b>	
Gompertz	53.2475	2.4324	1.5684	21.3476	22.1434	<b>4</b>	
Holling	50.6671	2.5166	1.5361	22.8012	23.9949	<b>3</b>	
<b>GLUECK</b>	3.6903	0.5792	0.2121	-12.0140	-12.0140	<b>1</b>	
<i>Leukemia</i> <a href="#">[23]</a>							
Logistic	223.7271	3.2640	1.6368	56.3235	58.5944	<b>2</b>	
Bertalanffy	273.6770	3.6992	1.6368	62.9585	66.3649	<b>5</b>	
Gompertz	259.9277	3.5182	1.6368	59.7729	62.0439	<b>4</b>	
Holling	248.5784	3.5255	1.6001	60.7461	64.1526	<b>3</b>	
<b>GLUECK</b>	35.2541	1.2381	0.3232	9.8230	9.8230	<b>1</b>	





**TUCANN**

Tumor Phenotypical Transitions **C**haracterization  
using **A**rtificial **N**eural **N**etworks

# Phenotypical transitions of tumors in DCIS

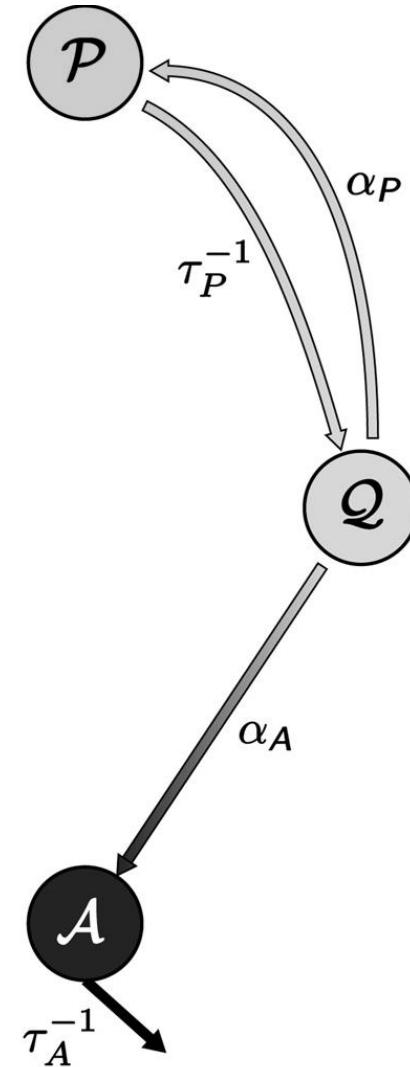
In typical cancer **phenotypic state space**, **quiescent** cancer cells (**Q**) can become **proliferative** (**P**) or **apoptotic** (**A**).

Can we learn **phenotypical transitions** from timeseries of raw immunohistochemistry and morphometric data?

$$\alpha_P = \frac{\frac{1}{\tau_P}(PI + PI^2) - \frac{1}{\tau_A}AIP I}{1 - AI - PI}$$

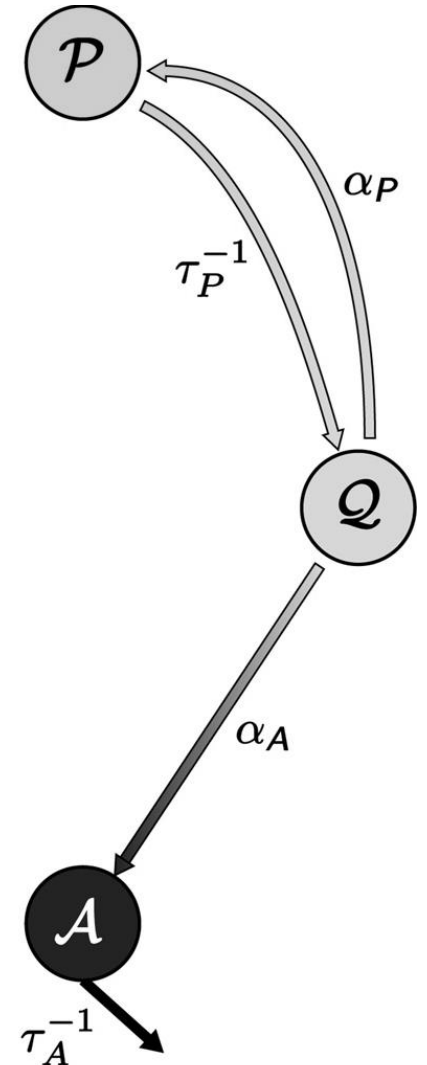
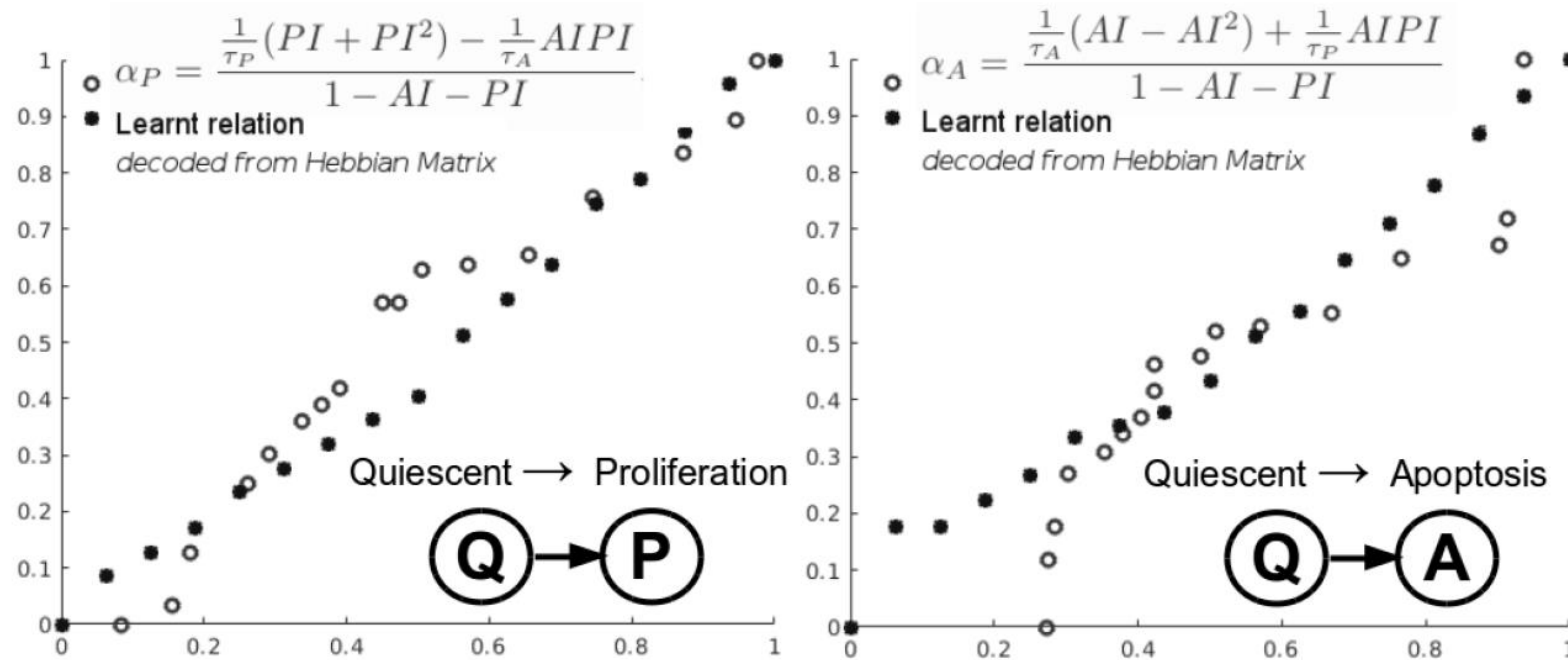
$$\alpha_A = \frac{\frac{1}{\tau_A}(AI - AI^2) + \frac{1}{\tau_P}AIP I}{1 - AI - PI}$$

where,  $\tau_P$  is the cells cycle time,  $\tau_A$  cells apoptosis time , PI proliferation index and AI apoptosis index.



# Phenotypical transitions of tumors in DCIS

Can we learn phenotypical transitions from timeseries of raw immunohistochemistry and morphometric data?





# CHIMERA

Combining Mechanistic Models and  
Machine Learning for Chemotherapy-  
Surgery Sequencing

# Formalizing therapy sequencing

**What is the best course of action for a particular patient, neoadjuvant or adjuvant chemotherapy?**

Assuming that the tumor size at time  $t_0 = 0$  is  $V_0$  there are two possible sequences:

- **Adjuvant chemotherapy:** At time  $t_0 > 0$  a fraction of the tumor is removed through surgery and subsequently chemotherapy is administered with a killing rate of  $1 - e^{-k_s}$  where  $k_s$  is a rate constant. The final size after the intervention, at  $t_f > t_0$  is  $V_{adj}$ .
- **Neoadjuvant chemotherapy:** At time  $t_0 > 0$  chemotherapy is administered with a predefined killing rate. At  $t_f > t_0$  a fraction  $1 - e^{-k_s}$  of the tumor is removed through surgery for a final size after the intervention  $V_{neoadj}$ .

The question of interest in our study is if  $V_{adj} > V_{neoadj}$ ?

# Formalizing therapy sequencing

If we consider  $f(V)$  the tumor growth model and  $P(t, V)$  the pharmacokinetics of the chemotherapeutic drug, we can formalize the two sequences as following:

- **Sequence 1:** Adjuvant setting, where size before surgery is  $\frac{dv_1}{dt} = f(v_1), v_1(0) = V_0, t \in [0, t_0]$  and size after surgery is

$$\frac{dV_1}{dt} = f(V_1) - P(t, V_1), V_1(t_0) = e^{-k_s} v_1(t_0), t \in [t_0, t_f].$$

In this case, the final volume of the tumor is  $V_{adj} = V_1(t_f)$ .

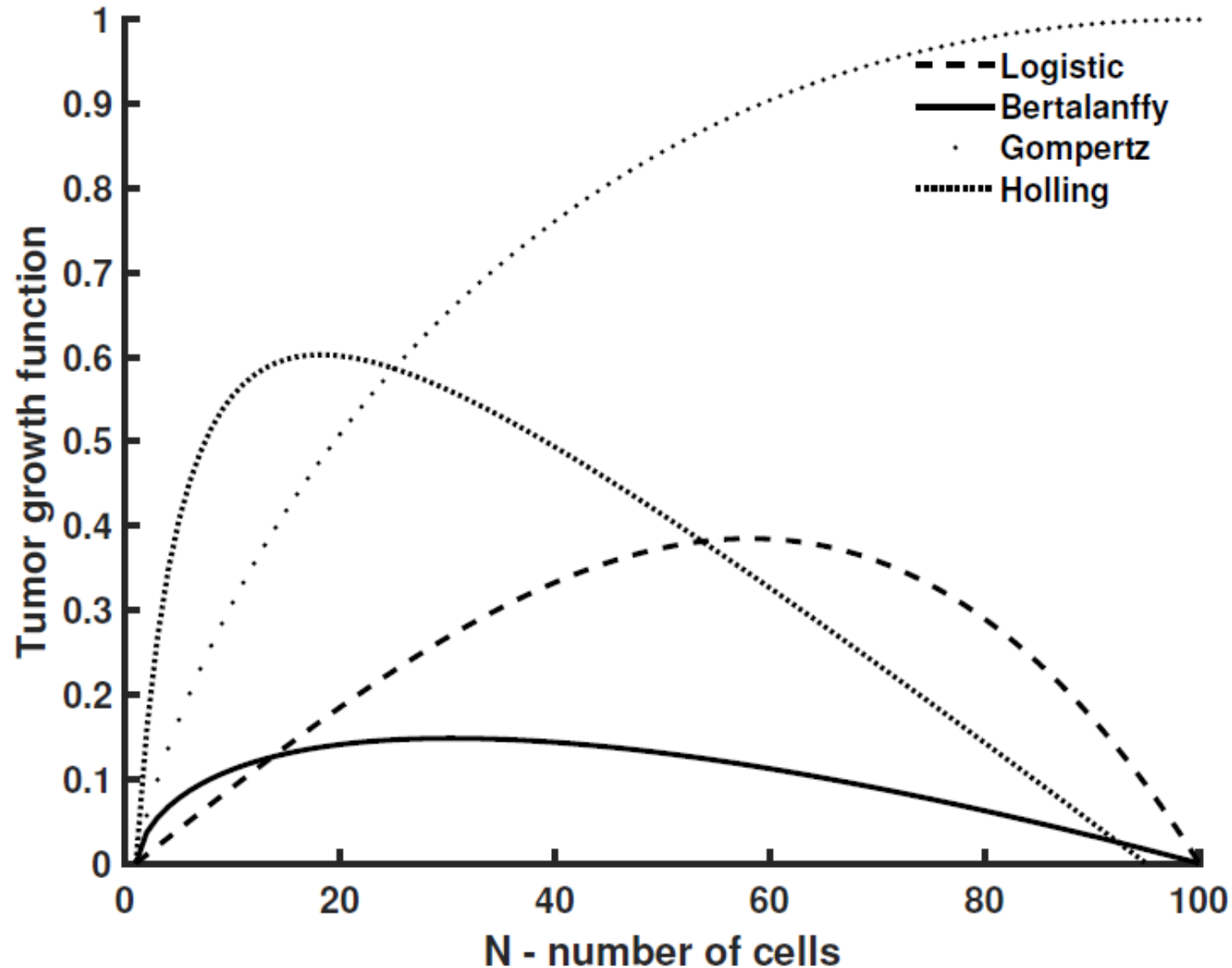
- **Sequence 2:** Neoadjuvant setting, where the size before chemotherapy onset is  $\frac{dv_2}{dt} = f(v_2), v_2(0) = V_0, t \in [0, t_0]$  and the size after chemotherapy onset is

$$\frac{dV_2}{dt} = f(V_2) - P(t, V_2), V_2(t_0) = v_2(t_0), t \in [t_0, t_f] \text{ respectively.}$$

Hence, for the neoadjuvant sequence, the final volume of the tumor is

$$V_{neoadj} = e^{-k_s} V_2(t_f).$$

# Tumor growth models



Model	Equation
Logistic	$\frac{dN}{dt} = \alpha N - \beta N^2$
Bertalanffy	$\frac{dN}{dt} = \alpha N^\lambda - \beta N$
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Parameters:

$N$  - cell population size (or volume),  
 $\alpha$  - growth rate,  
 $\beta$  - cell death rate,  
 $\lambda$  - nutrient limited proliferation rate,  
 $k$  - carrying capacity of cells.

# Pharmacokinetics models

In our study, we use the data from the computational model of **Paclitaxel pharmacokinetics** of Kuh et al. 2000 [8], due to its wide use in **breast cancer chemotherapy schemes**.

The model describes the factors that determine the kinetics of **Paclitaxel uptake, binding, and efflux** from cells

$$\frac{dc(t)}{dt} = \left[ \frac{-A + \sqrt{A^2 + 4K_{d,m}c_m(t)}}{2} - \frac{-B + \sqrt{B^2 + 4(1 + NSB)K_{d,c}c(t)}}{2(1 + NSB)} \right] \frac{CL_f}{V_{onecell}} - k_{cellnumber}c(t)$$

where:

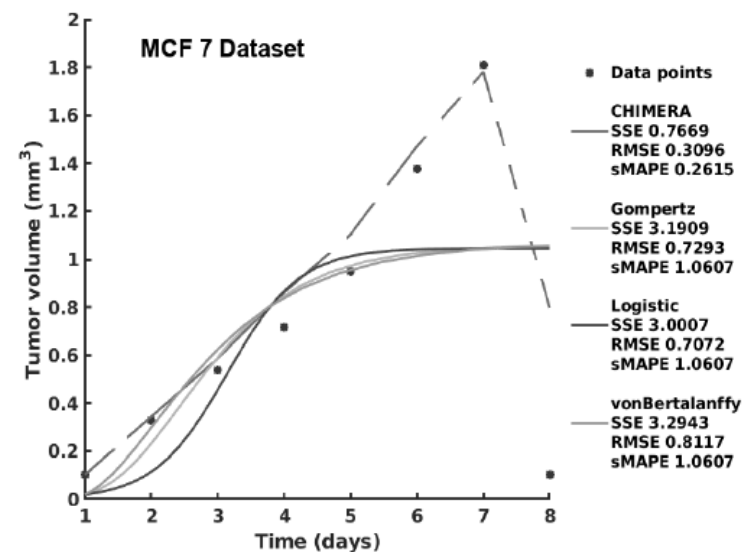
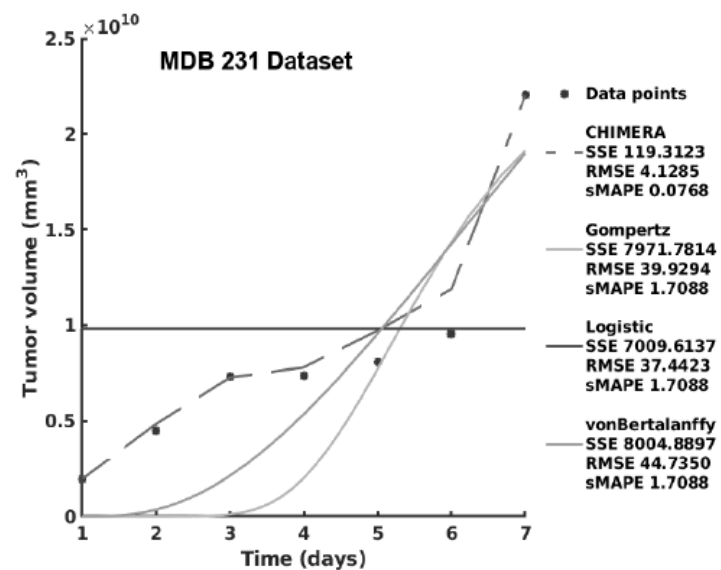
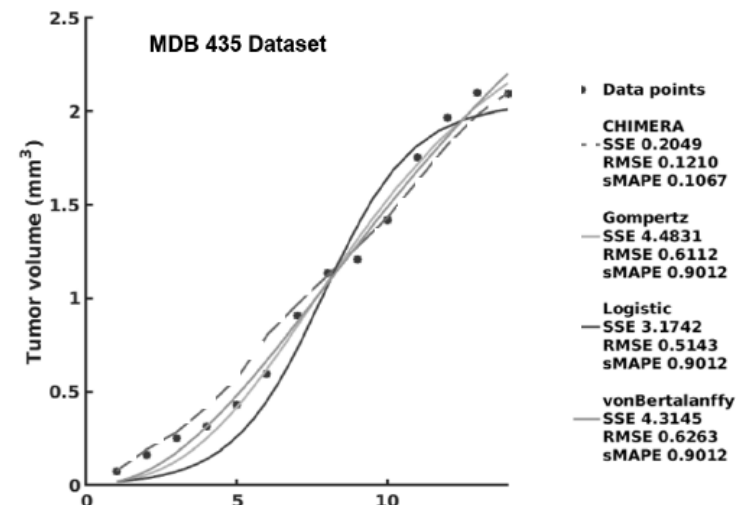
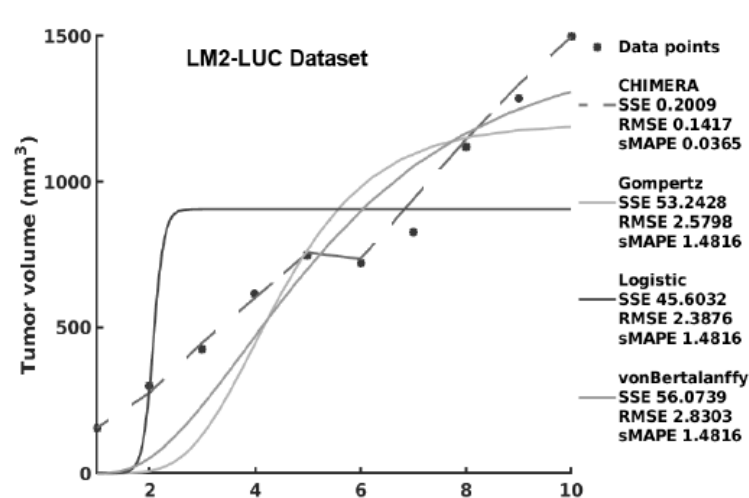
- $V_{onecell}$  is the average cell volume
- $ICN$  is the initial cell number
- $NSB$  is the proportionality constant for non-saturable binding sites in cells
- $k_{cellnumber}$  is the rate constant for changes in cell number
- $A$  is a function of the constant for drug binding to proteins in medium  $K_{d,m}$
- $B$  is a function of the constant for drug binding to proteins in cells
- $CL_f$  is the clearance of free drug by passive diffusion, on a per cell basis
- $c_m$  concentration of drug in the medium, calculated as:

$$\frac{dc_m(t)}{dt} = \left[ \frac{-A + \sqrt{A^2 + 4K_{d,m}c_m(t)}}{2} - \frac{-B + \sqrt{B^2 + 4(1 + NSB)K_{d,c}c(t)}}{2(1 + NSB)} \right] \frac{CL_f ICN e^{k_{cellnumber}t}}{V_m}$$



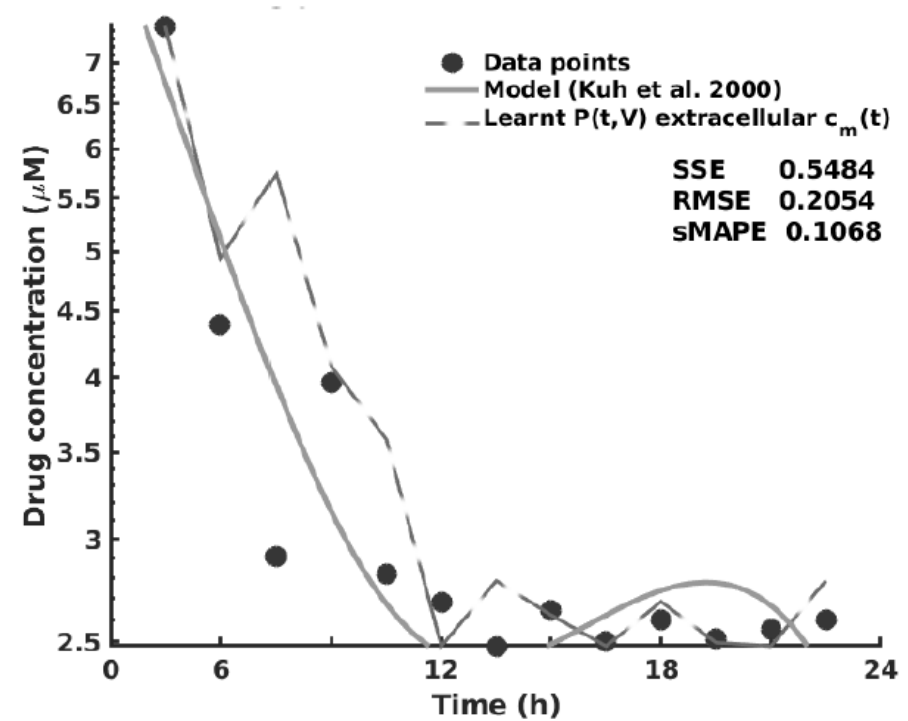
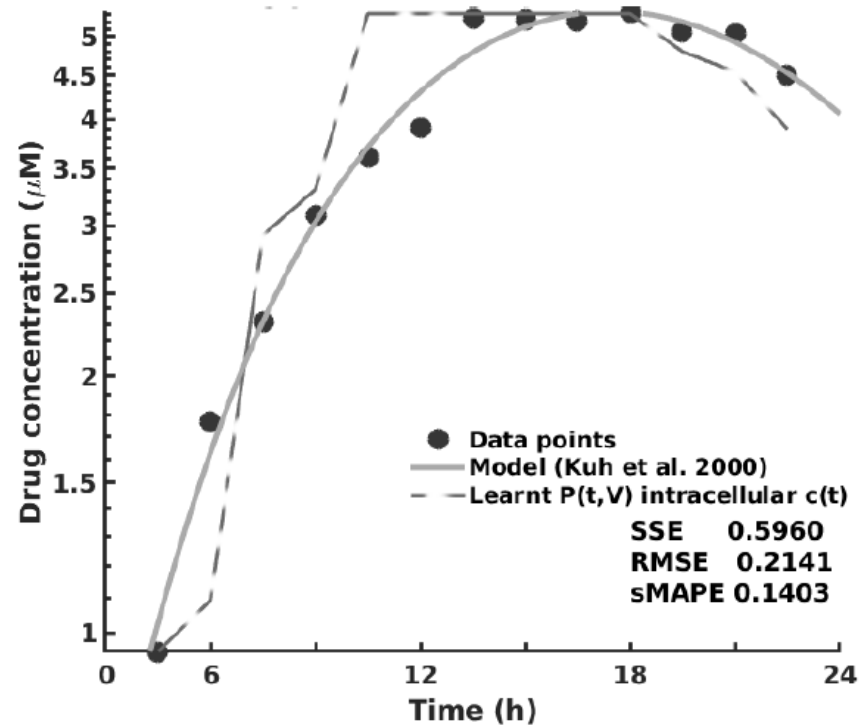
# Experiments and evaluation

## *Learning tumor growth*



# Experiments and evaluation

## *Learning pharmacokinetics*



# Experiments and evaluation

## *Chemotherapy-Surgery Sequencing*

Following our initial derivation,  $V_{neoadj} = e^{-k_s} V_2(t_f)$  and  $V_{adj} = V_1(t_f)$  correspond to tumor sizes in neo-adjuvant and adjuvant sequences, respectively.

Under the **log-kill assumption**, if we let  $c(t) = -\int_{t_0}^{t_f} c(s) e^{\beta s} ds$  then

$$\frac{V_{neoadj}}{V_{adj}} = \exp\{-k_s (1 - e^{-\beta(t_f - t_0)})\} < 1$$

*A given dose of chemotherapy kills the same fraction of tumor cells regardless of the size of the tumor at the time of treatment.*

hence  $V_{neoadj} < V_{adj}$ .

Similarly, under the **Norton-Simon assumption** we obtain

$$\frac{V_{neoadj}}{V_{adj}} = \exp\{-k_s (1 - e^{-\beta(t_f - t_0) + c(t_f)})\}$$

*The rate of cancer cell death in response to treatment is directly proportional to the tumor growth rate at the time of treatment.*

which for  $c(t) = \int_{t_0}^{t_f} c(s) ds < t_f - t_0$  determines  $V_{neoadj} < V_{adj}$ .

# Experiments and evaluation

## *Chemotherapy-Surgery Sequencing*

Let's consider the **MCF-7 cell line dataset** from Tan et al. 2015 [16] described in our Experimental setup. We use the derivations for  $V_{neoadj}$  and  $V_{adj}$  and fill in with the **decoded values** from the **learnt tumor growth**  $f(V)$  and **learnt pharmacokinetics**  $P(t, V)$ .

Model		
(Biological Parameters)	Log-kill hypothesis	Norton-Simon hypothesis
Gompertz ( $\beta, K, v$ )	$V_{neoadj} < V_{adj}$	$V_{neoadj} < V_{adj}^*$
CHIMERA (none)	$V_{neoadj} < V_{adj}$	$V_{neoadj} > V_{adj}$

---

\* Holds only if  $c(t) = \int_{t_0}^{t_f} c(s)ds < t_f - t_0$ .

**CHIMERA** uses **learnt tumor growth** and **pharmacokinetics** to **infer** the most appropriate **sequence of therapy**, **consistent with its mechanistic counterparts**, but **without** extensive biological **parametrization**.



**PERFECTO**

Prediction of **E**xtended **R**esponse and Growth  
Functions for **E**stimating **C**hemotherapy **O**utcomes

# Chemotherapy regimen planning

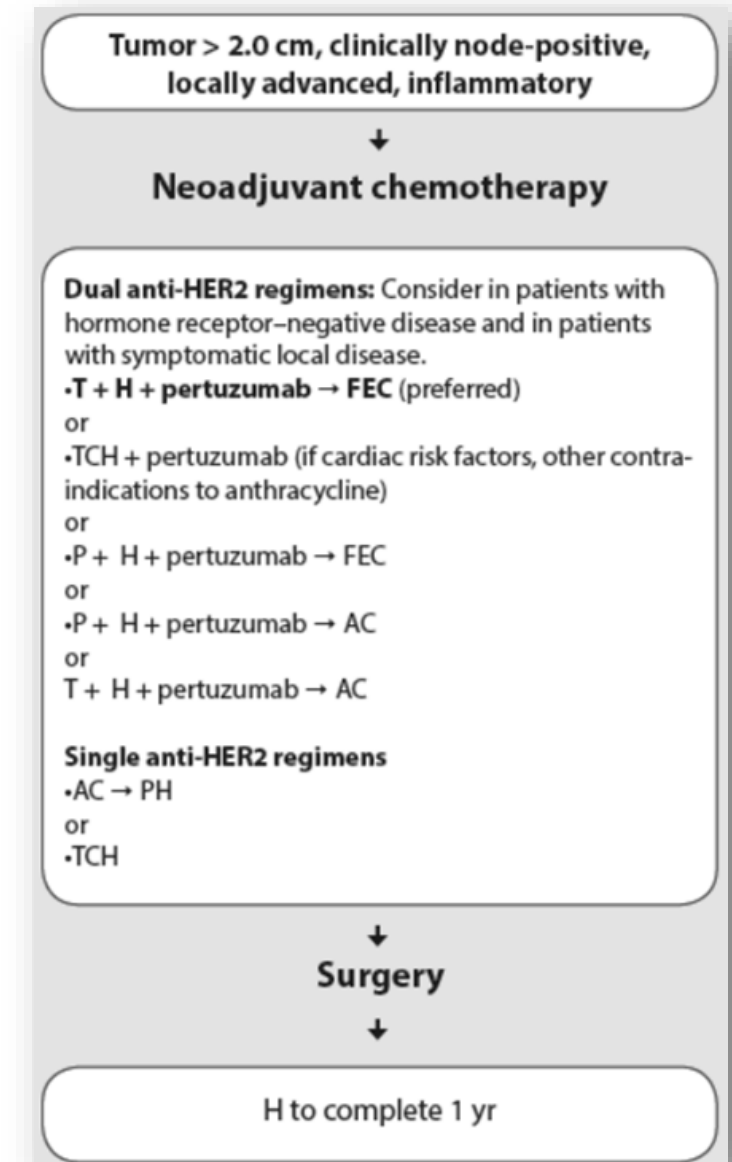
## Context

**Chemotherapy regimens** are chosen primarily based on:

- empirical **data** from **clinical trials**
- patient's **form** and **subtype** of **cancer**
- **progression** to metastases
- **high-risk indications**
- **prognosis**

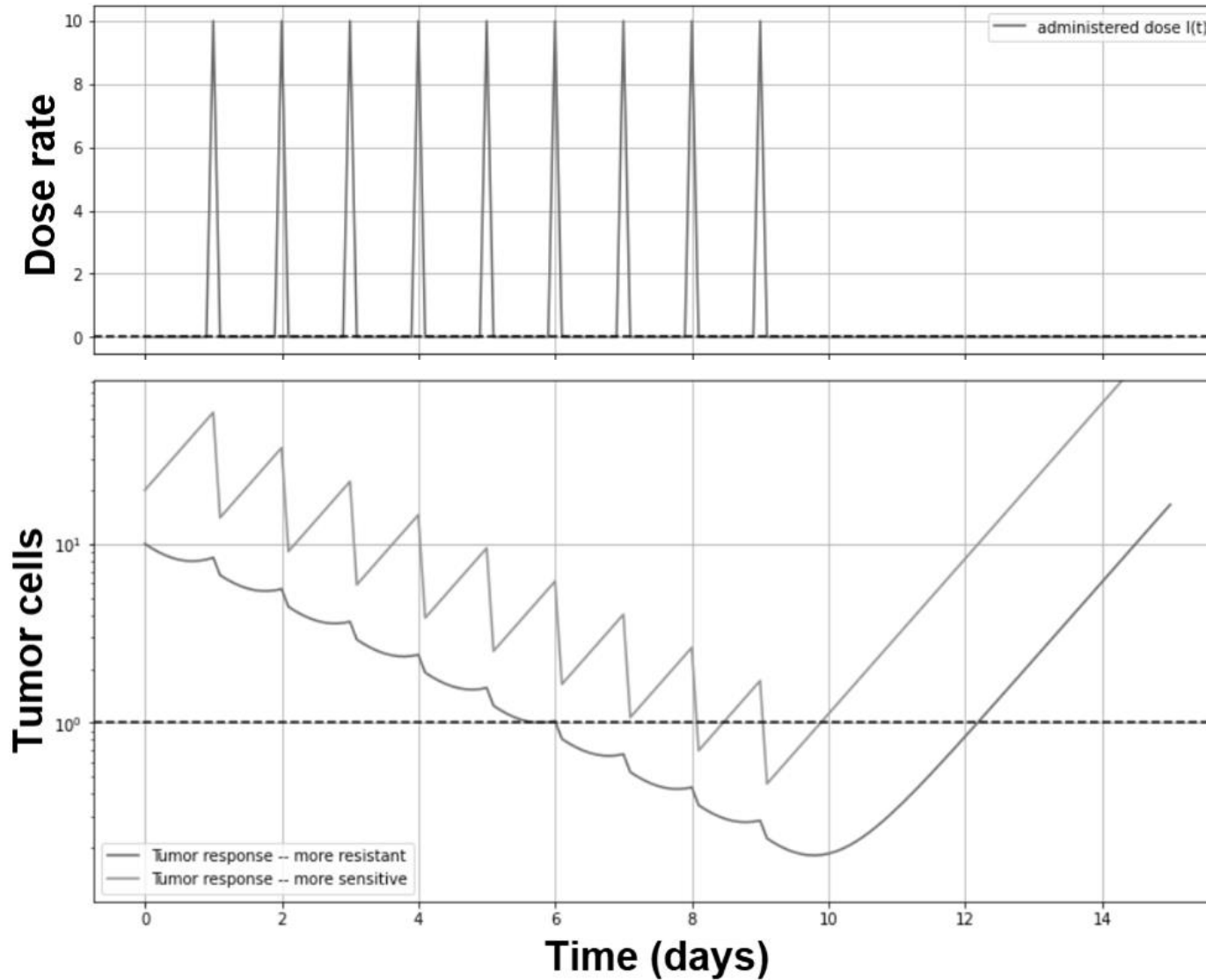
## Problem

Challenges in successfully **predicting the effectiveness** (i.e. size of the tumor after **neoadjuvant chemotherapy**) of any particular chemotherapy plan for any given patient **before the initiation of the regimen**.

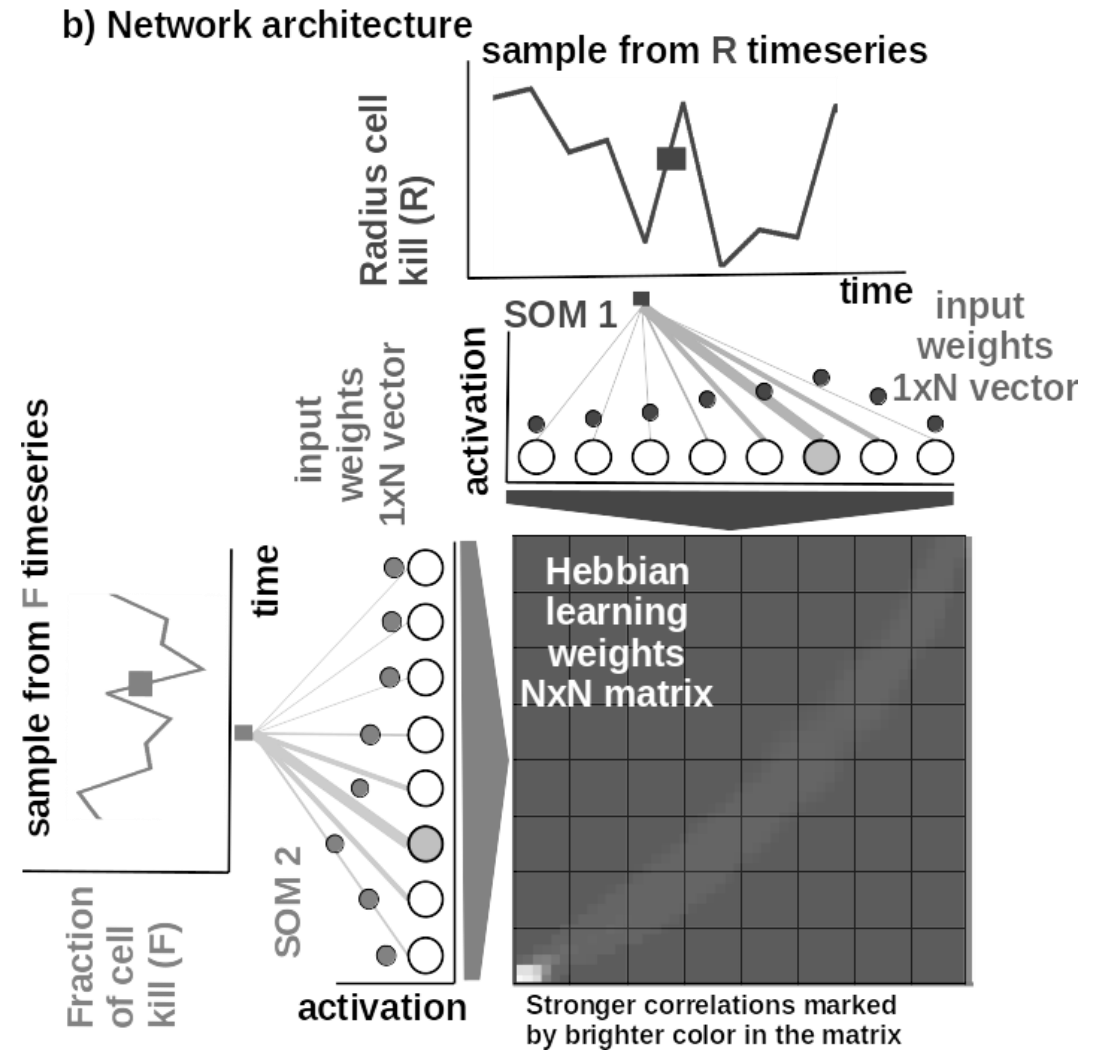
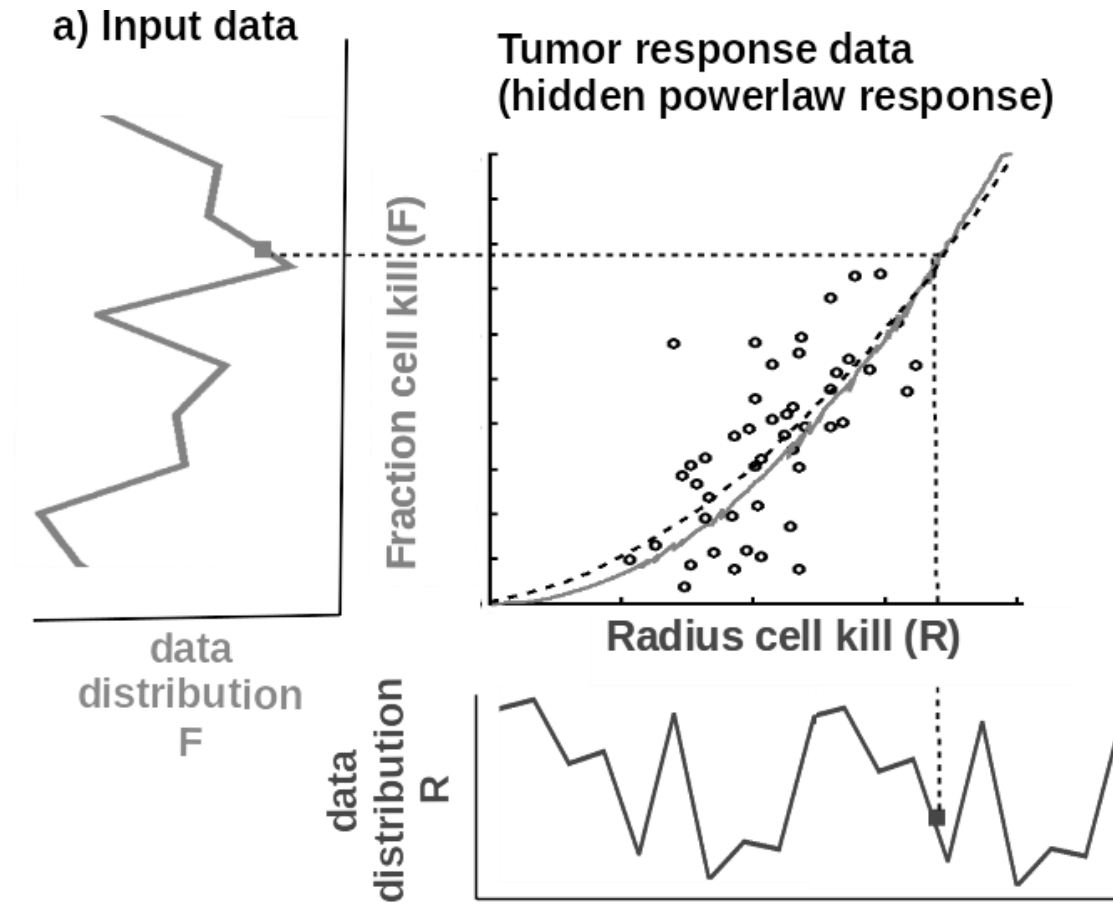


# Tumor growth models

## *Growth under chemotherapy*



# Model instantiation



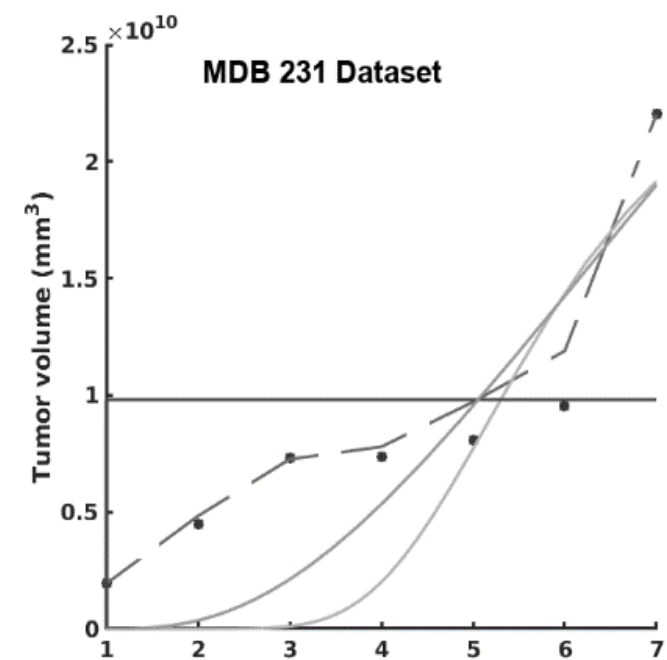
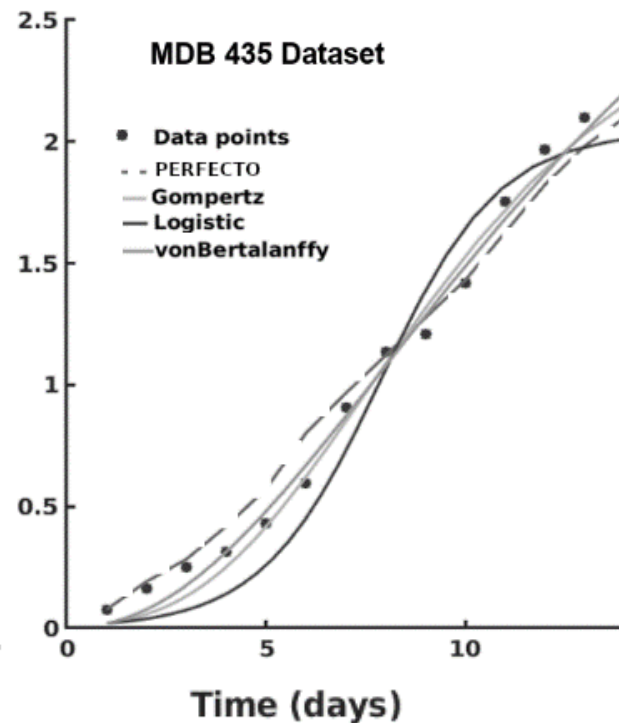
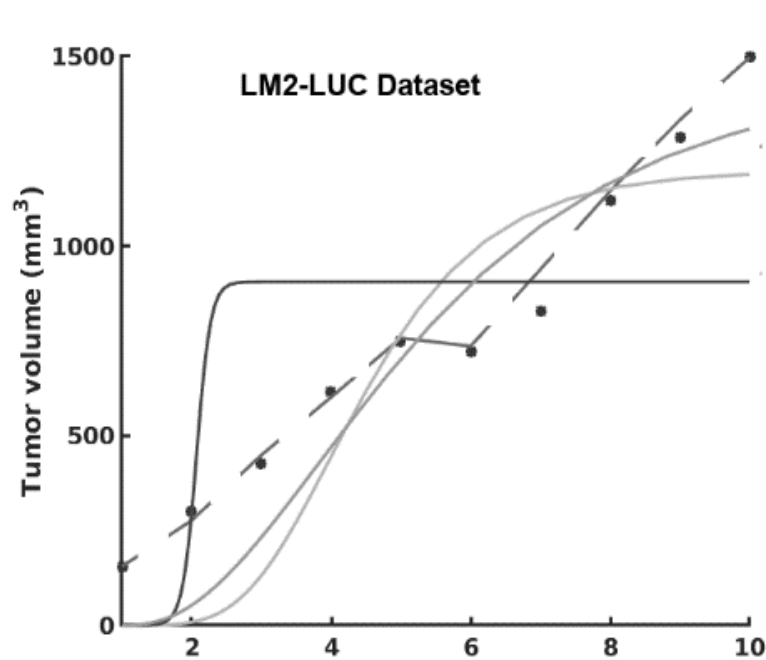


# Experiments and evaluation

*Learning unperturbed tumor growth*

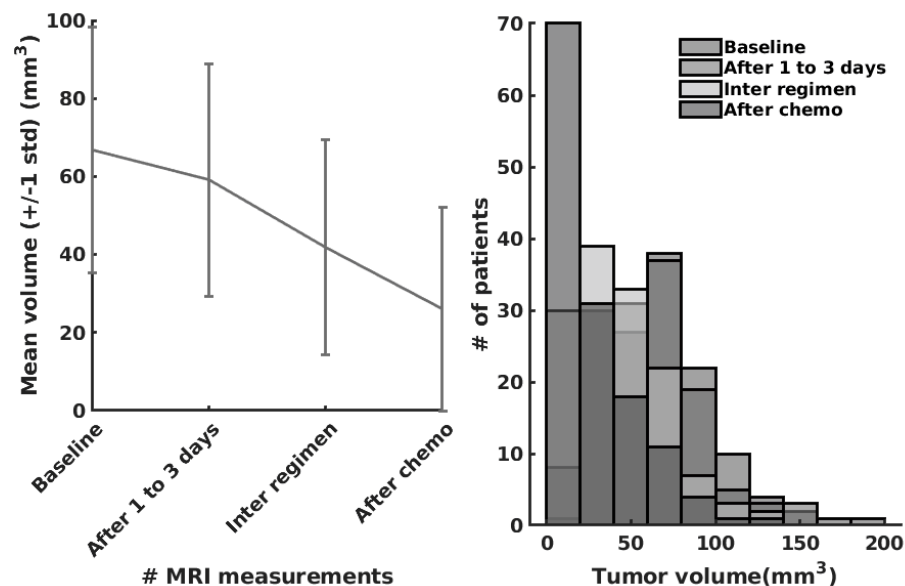
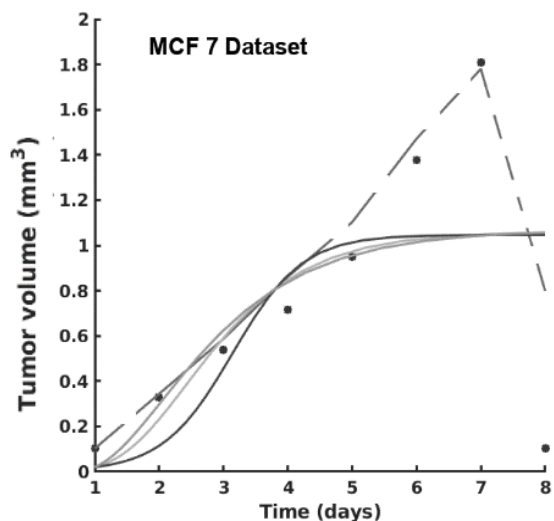
Experimental dataset setup

Dataset	Cancer Type	Data Type	Data Points	Data Freq.
1	MDA-MB-231 cell line	Fluorescence imaging	7	2x/week
2	MDA-MB-435 cell line	Digital Caliper	14	2x/week
3	MCF-7 cell line	Caliper	8	1x/week
4	LM2-4LUC+ cell line	Digital Caliper	10	3x/week



# Experiments and evaluation

## Learning perturbed tumor growth



Dataset/Model	SSE	RMSE	sMAPE
<i>MDA-MB-231 cell line cancer [29]</i>			
Logistic	7009.6	37.4423	1.7088
Bertalanffy	8004.9	44.7350	1.7088
Gompertz	7971.8	39.9294	1.7088
<b>PERFECTO</b>	119.3	4.1285	0.0768

<i>MDA-MB-435 cell line cancer [15]</i>			
Logistic	0.2936	0.1713	0.1437
Bertalanffy	0.2315	0.1604	0.1437
Gompertz	0.3175	0.1782	0.1437
<b>PERFECTO</b>	0.0977	0.0902	0.0763

<i>MCF-7 cell line cancer [30]</i>			
Logistic	3.0007	0.7072	1.0607
Bertalanffy	3.2943	0.8117	1.0607
Gompertz	3.1909	0.7293	1.0607
<b>PERFECTO</b>	0.7669	0.3096	0.2615

<i>LM2-4LUC+ cell line cancer [31]</i>			
Logistic	45.6032	2.3876	1.4816
Bertalanffy	56.0739	2.8303	1.4816
Gompertz	53.2428	2.5798	1.4816
<b>PERFECTO</b>	0.2009	0.1417	0.0365

<i>I-SPY2 Trial [32]</i>			
Logistic	248.3735	11.1439	1.7833
Bertalanffy	259.0963	16.0963	1.7834
Gompertz	260.3747	11.4100	1.7883
<b>PERFECTO</b>	0.8650	0.4650	0.0389

AKII Lab

# AKII Lab Team



DR. CRISTIAN  
AXENIE,  
GROUP LEADER,  
PI IN AI AND ML



PROF. DR. THOMAS  
GRAUSCHOPF,  
PI IN VR



GHEORGHE LISCA,  
PHD STUDENT



XIAORUI DU,  
PHD STUDENT

## Helios Klinikum München West

Akademisches Lehrkrankenhaus der  
Ludwig-Maximilians-Universität München

### Daria Kurz

Leitende Oberärztin

Gynäkologisches  
Krebszentrum

Interdisziplinäres  
Brustzentrum



CRISTOBAL  
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BA STUDENT



ARMIN  
BECHER,  
RESEARCH ASSISTANT



SEBASTIAN  
POHL,  
MSC STUDENT



STEFAN  
SCHIECHEL,  
BA STUDENT



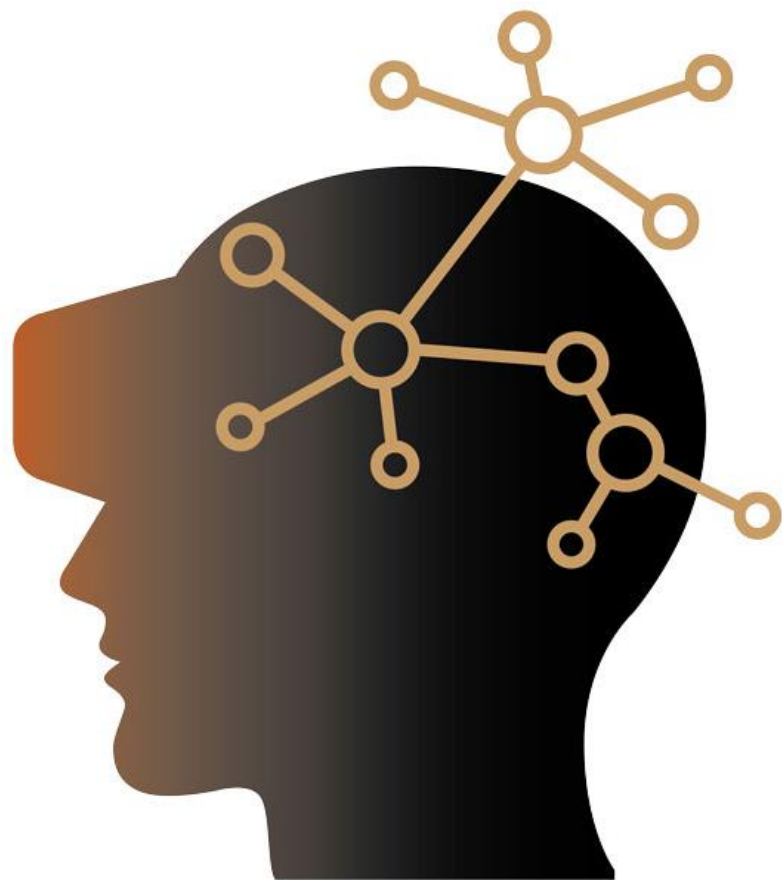
MARTIN  
KUNZ,  
BA STUDENT

# AKII Lab Origins

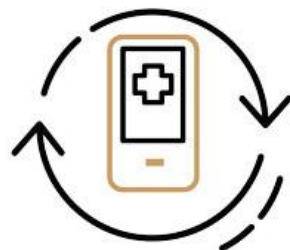




# AKII Lab Profile



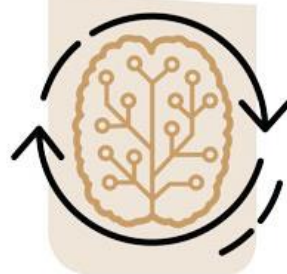
Biomedical  
Engineering



Computational  
Oncology



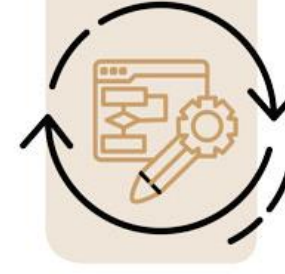
Biomechanics  
Sport and Rehabilitation



Machine  
Learning



Virtual  
Reality



Sensors